Solutions to Problem 1.

a. $$\begin{aligned}\Pr\{Y_4 > 30 \mid Y_2 = 10\} &= \Pr\{Y_4 - Y_2 > 20 \mid Y_2 = 10\} \\ &= \Pr\{Y_4 - Y_2 > 20\} \\ &= \Pr\{Y_2 > 20\} \\ &= 1 - \Pr\{Y_2 \le 20\} \\ &= 1 - \sum_{j=0}^{20} \frac{e^{-8(2)}(8(2))^j}{j!} \approx 0.1318\end{aligned}$$

b. $\Pr\{T_{50} \le 6\} = F_{T_{50}}(6)$ (T_{50} is Erlang distributed with $n = 50$ phases and parameter $\lambda = 8$)

$$= 1 - \sum_{j=0}^{49} \frac{e^{-8(6)}(8(6))^j}{j!} \approx 0.405$$

Note. You should get the same answer if you computed $\Pr\{Y_6 \ge 50\}$ instead.

c. $$\begin{aligned}\Pr\{T_{100} \le 12 \mid Y_6 = 40\} &= \Pr\{Y_{12} \ge 100 \mid Y_6 = 40\} \\ &= \Pr\{Y_{12} - Y_6 \ge 60 \mid Y_6 = 40\} \\ &= \Pr\{Y_{12} - Y_6 \ge 60\} \\ &= \Pr\{Y_6 \ge 60\} \\ &= 1 - \sum_{j=0}^{59} \frac{e^{-8(6)}(8(6))^j}{j!} \approx 0.0523\end{aligned}$$

d. $E[T_4] = \frac{4}{8} = \frac{1}{2}$